

The answers to the following are in agreement with the text except for the yellow cells below, and for those particular cases the text answer has the suggestion of a typo.

11 - 20 Multiplication, addition, and transposition of matrices and vectors

$$\mathbf{A} = \begin{pmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{pmatrix}$$

`{{4, -2, 3}, {-2, 1, 6}, {1, 2, 2}}`

$$\mathbf{B} = \begin{pmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

`{{1, -3, 0}, {-3, 1, 0}, {0, 0, -2}}`

$$\mathbf{CC} = \begin{pmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{pmatrix}$$

`{{0, 1}, {3, 2}, {-2, 0}}`

$$\mathbf{ar} = \{1, -2, 0\}$$

`{1, -2, 0}`

$$\mathbf{bc} = \{\{3\}, \{1\}, \{-1\}\} // \text{MatrixForm}$$

$$\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

$$\mathbf{bcr} = \{3, 1, -1\} // \text{MatrixForm}$$

$$\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

Showing all intermediate results, calculate the following expressions or give reasons why they are undefined:

11. \mathbf{AB} , \mathbf{AB}^T , \mathbf{BA} , $\mathbf{B}^T\mathbf{A}$

$$\mathbf{A.B} // \text{MatrixForm}$$

$$\begin{pmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{pmatrix}$$

A.B[†] // MatrixForm

$$\begin{pmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{pmatrix}$$

B.A // MatrixForm

$$\begin{pmatrix} 10 & -5 & -15 \\ -14 & 7 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

Above: The result shown is not in agreement with the text answer, which has -33 for a_{23} (instead of -3).

B[†].A // MatrixForm

$$\begin{pmatrix} 10 & -5 & -15 \\ -14 & 7 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

Above: Since the answer block lists 'same' for this result, the answer is again at odds (technically) with the text.

13. CC^T , BC, CB, C^TB

CC.CC^T // MatrixForm

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 13 & -6 \\ 0 & -6 & 4 \end{pmatrix}$$

B.CC // MatrixForm

$$\begin{pmatrix} -9 & -5 \\ 3 & -1 \\ 4 & 0 \end{pmatrix}$$

CC.B // MatrixForm

Dot:dotsh Tensor{{0, 1}, {3, 2}, {-2, 0}} and {{1, -3, 0}, {-3, 1, 0}, {0, 0, -2}} have incompatible shapes >>

{{0, 1}, {3, 2}, {-2, 0}}.{{1, -3, 0}, {-3, 1, 0}, {0, 0, -2}}

CC^T.B // MatrixForm

$$\begin{pmatrix} -9 & 3 & 4 \\ -5 & -1 & 0 \end{pmatrix}$$

The operations above agree with the text answers.

15. Aa , Aa^T , $(Ab)^T$, b^TA^T

I'm going to redefine the vectors so they are clearly visible.

```
ar = {{1, -2, 0}}
{{1, -2, 0}}
```

```
bc = {{3}, {1}, {-1}}
{{3}, {1}, {-1}}
```

Mathematica can do a dot product between matrix and vector. However, it does not distinguish between row vectors and column vectors.

```
Dot[A, ar]
```

Dot::dots: Tensors{{4, -2, 3}, {-2, 1, 6}, {1, 2, 2}} and {{1, -2, 0}} have incompatible shapes >>

```
{{4, -2, 3}, {-2, 1, 6}, {1, 2, 2}}.{{1, -2, 0}}
```

Above: *Mathematica* returns the input to show it cannot perform the first operation, which agrees with the text answer assessment.

```
Dot[A, bc]
```

```
{{7}, {-11}, {3}}
```

Above: *Mathematica* formed a dot product with what were disguised as a row vector, then a column vector.

```
cja = Transpose[ar]
```

```
{{1}, {-2}, {0}}
```

Above: Transposing a row vector in *Mathematica* does work.

```
cjb = Transpose[bc]
```

```
{{3, 1, -1}}
```

Above: Transposing a column vector does work in *Mathematica*.

```
Dot[A, cja]
```

```
{{8}, {-4}, {-3}}
```

Above: This is the second listed operation, and agrees with the text.

```
inter = Dot[A, bc]
```

```
{{7}, {-11}, {3}}
```

```
fin = Transpose[inter]
```

```
{{7, -11, 3}}
```

Above: This is the third-listed operation, and *Mathematica* produces an answer. The answer does not agree with the text answer, though it looks good to me. The text answer is {{7, -1, 3}}.

Dot[cjb, Transpose[A]]

```
{7, -11, 3}
```

Above: This is the 4th-listed operation. The text answer states that the result is the same as on the third operation, which, if true, means there is a disagreement with the Mathematica answer.

17. ABC, ABa, ABb, Ca^T

A.B.CC // MatrixForm

$$\begin{pmatrix} -30 & -18 \\ 45 & 9 \\ 5 & -7 \end{pmatrix}$$

Above: The answer in green agrees with the text answer.

A.B.ar

```
Dot::dotsh: Tensors{{10, -14, -6}, {-5, 7, -12}, {-5, -1, -4}} and {{1, -2, 0}} have incompatible shapes >>
```

```
{{10, -14, -6}, {-5, 7, -12}, {-5, -1, -4}}.{{1, -2, 0}}
```

Above: Mathematica agrees with the text that the operation called for is undefined.

A.B.bc

```
{{22}, {4}, {-12}}
```

Above: Mathematica's answer agrees with the text answer.

Dot[CC, cja]

```
Dot::dotsh: Tensors{{0, 1}, {3, 2}, {-2, 0}} and {{1}, {-2}, {0}} have incompatible shapes >>
```

```
{{0, 1}, {3, 2}, {-2, 0}}.{{1}, {-2}, {0}}
```

Above: Undefined as stated in the text.

19. $1.5a+3.0b, 1.5a^T + 3.0b, (A - B)b, Ab - Bb$

1.5 ar + 3.0 bc // MatrixForm

```
Thread::tdlerr: Objects of unequal length in {{1.5, -3., 0.}} + {{9.}, {3.}, {-3.}} cannot be combined >>
```

```
{{1.5, -3., 0.}} + {{9.}, {3.}, {-3.}}
```

1.5 cja + 3 bc // MatrixForm

$$\begin{pmatrix} 10.5 \\ 0. \\ -3. \end{pmatrix}$$

The answer in the green cell above matches the text answer.

(A - B) . bc

{{7}, {-3}, {1}}

The answer in the green cell above matches the text answer.

A.bc - B.bc

{{7}, {-3}, {1}}

The answer in the green cell above matches the text answer.

Take home thoughts from this section. Mathematica is very compliant and free-wheeling with regard to vectors. In order to get expected results where it is necessary to distinguish between row vectors and column vectors, row vectors should be entered as `{{a, b, c}}`, and column vectors as `{{a}, {b}, {c}}`.